

March 12, 2018

Centralne razlike

x	f	Δf	$\Delta^2 f$	$\Delta^3 f$	$\Delta^4 f$	$\Delta^5 f$	$\Delta^6 f$
\vdots	\vdots	\vdots					
X_{-4}	f_{-4}	\vdots					
		Δf_{-4}	\vdots				
X_{-3}	f_{-3}		$\Delta^2 f_{-4}$	\vdots			
		Δf_{-3}		$\Delta^3 f_{-4}$	\vdots		
X_{-2}	f_{-2}		$\Delta^2 f_{-3}$		$\Delta^4 f_{-4}$	\vdots	
		Δf_{-2}		$\Delta^3 f_{-3}$		$\Delta^5 f_{-4}$	\vdots
X_{-1}	f_{-1}		$\Delta^2 f_{-2}$		$\Delta^4 f_{-3}$		$\Delta^6 f_{-4}$
		Δf_{-1}		$\Delta^3 f_{-2}$		$\Delta^5 f_{-3}$	
X_0	f_0		$\Delta^2 f_{-1}$		$\Delta^4 f_{-2}$		$\Delta^6 f_{-3}$
		Δf_0		$\Delta^3 f_{-1}$		$\Delta^5 f_{-2}$	
X_1	f_1		$\Delta^2 f_0$		$\Delta^4 f_{-1}$		$\Delta^6 f_{-2}$
		Δf_1		$\Delta^3 f_0$		$\Delta^5 f_{-1}$	\vdots
X_2	f_2		$\Delta^2 f_1$		$\Delta^4 f_0$	\vdots	
		Δf_2		$\Delta^3 f_1$	\vdots		
X_3	f_3		$\Delta^2 f_2$	\vdots			
		Δf_3	\vdots				
X_4	f_4	\vdots					
\vdots	\vdots						

$$x \in (x_0, x_0 + \frac{h}{2}]$$

Redosled čvorova: $x_0, x_1, x_{-1}, \dots, x_n, x_{-n}, x_{n+1}$

$$\begin{aligned} L_{2n+1}(x) &= f(x_0) + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_{-1}](x - x_0)(x - x_1) \\ &\quad + f[x_0, x_1, x_{-1}, x_2](x - x_0)(x - x_1)(x - x_{-1}) + \dots \\ &\quad + f[x_0, x_1, \dots, x_{-n}, x_{n+1}](x - x_0) \cdot \dots \cdot (x - x_n)(x - x_{-n}) \end{aligned}$$

$$\begin{aligned} L_{2n+1}(x) &= f(x_0) + q\Delta f_0 + \frac{q(q-1)}{2!} \Delta^2 f_{-1} + \frac{q(q^2-1)}{3!} \Delta^3 f_{-1} + \dots \\ &\quad + \frac{q(q^2-1) \cdot \dots \cdot (q^2-n^2)}{(2n+1)!} \Delta^{2n+1} f_{-n} \end{aligned}$$

x	f	Δf	$\Delta^2 f$	$\Delta^3 f$	$\Delta^4 f$	$\Delta^5 f$	$\Delta^6 f$...
\vdots	\vdots	\vdots						
x_{-4}	f_{-4}	Δf_{-4}	\vdots					
x_{-3}	f_{-3}		$\Delta^2 f_{-4}$	\vdots				
x_{-2}	f_{-2}		$\Delta^2 f_{-3}$	$\Delta^3 f_{-4}$	\vdots			
x_{-1}	f_{-1}	Δf_{-2}	$\Delta^2 f_{-2}$	$\Delta^3 f_{-3}$	$\Delta^4 f_{-4}$	\vdots		
x_0	f_0	Δf_0	$\Delta^2 f_{-1}$	$\Delta^3 f_{-2}$	$\Delta^4 f_{-2}$	$\Delta^5 f_{-3}$	$\Delta^6 f_{-3}$	
x_1	f_1	Δf_1	$\Delta^2 f_0$	$\Delta^3 f_0$	$\Delta^4 f_{-1}$	$\Delta^5 f_{-2}$	$\Delta^6 f_{-2}$...
x_2	f_2	Δf_2	$\Delta^2 f_1$	$\Delta^3 f_1$	$\Delta^4 f_0$	\vdots		
x_3	f_3		$\Delta^2 f_2$	\vdots				
x_4	f_4	\vdots						

$$x \in [x_0 + \frac{h}{2}, x_1)$$

Redosled čvorova: $x_1, x_0, x_2, x_{-1}, \dots, x_{n+1}, x_{-n}$

$$\begin{aligned} L_{2n+1}(x) &= f(x_1) + f[x_1, x_0](x - x_1) + f[x_1, x_0, x_2](x - x_1)(x - x_0) \\ &\quad + f[x_1, x_0, x_2, x_{-1}](x - x_1)(x - x_0)(x - x_2) + \dots \\ &\quad + f[x_1, x_0, \dots, x_{n+1}, x_{-n}](x - x_1) \cdot \dots \cdot (x - x_{-(n-1)})(x - x_{n+1}) \end{aligned}$$

$$\begin{aligned} L_{2n+1}(x) &= f(x_1) + (q-1)\Delta f_0 + \frac{q(q-1)}{2!} \Delta^2 f_0 + \frac{q(q-1)(q-2)}{3!} \Delta^3 f_{-1} + \dots \\ &\quad + \frac{q(q^2-1) \cdot \dots \cdot (q^2 - (n-1)^2)(q-n)(q-(n+1))}{(2n+1)!} \Delta^{2n+1} f_{-n} \end{aligned}$$

x	f	Δf	$\Delta^2 f$	$\Delta^3 f$	$\Delta^4 f$	$\Delta^5 f$	$\Delta^6 f$...
\vdots	\vdots	\vdots						
x_{-4}	f_{-4}	Δf_{-4}	\vdots					
x_{-3}	f_{-3}		$\Delta^2 f_{-4}$	\vdots				
x_{-2}	f_{-2}		$\Delta^2 f_{-3}$	$\Delta^3 f_{-4}$	\vdots			
x_{-1}	f_{-1}	Δf_{-2}	$\Delta^2 f_{-2}$	$\Delta^3 f_{-3}$	$\Delta^4 f_{-4}$	\vdots		
x_0	f_0	Δf_0	$\Delta^2 f_{-1}$	$\Delta^3 f_{-2}$	$\Delta^4 f_{-3}$	$\Delta^5 f_{-4}$	$\Delta^6 f_{-4}$	
x_1	f_1	Δf_1	$\Delta^2 f_0$	$\Delta^3 f_{-1}$	$\Delta^4 f_{-2}$	$\Delta^5 f_{-2}$	$\Delta^6 f_{-3}$	
x_2	f_2	Δf_2	$\Delta^2 f_1$	$\Delta^3 f_0$	$\Delta^4 f_0$	$\Delta^5 f_{-1}$	\vdots	
x_3	f_3		$\Delta^2 f_2$	\vdots				
x_4	f_4	\vdots						

$$x \in [x_0 - \frac{h}{2}, x_0)$$

Redosled čvorova: $x_0, x_{-1}, x_1, x_{-2}, x_2, \dots, x_n, x_{-(n+1)}$

$$\begin{aligned} L_{2n+1}(x) &= f(x_0) + f[x_0, x_{-1}](x - x_0) + f[x_0, x_{-1}, x_1](x - x_0)(x - x_{-1}) \\ &\quad + f[x_0, x_{-1}, x_1, x_{-2}](x - x_0)(x - x_{-1})(x - x_1) + \dots \\ &\quad + f[x_0, x_{-1}, \dots, x_n, x_{-(n+1)}](x - x_0) \cdot \dots \cdot (x - x_{-n})(x - x_n) \end{aligned}$$

$$\begin{aligned} L_{2n+1}(x) &= f(x_0) + q\Delta f_{-1} + \frac{q(q+1)}{2!} \Delta^2 f_{-1} + \frac{q(q^2-1)}{3!} \Delta^3 f_{-2} + \dots \\ &\quad + \frac{q(q^2-1) \cdot \dots \cdot (q^2-n^2)}{(2n+1)!} \Delta^{2n+1} f_{-(n+1)} \end{aligned}$$

Aritmetička sredina prvog i treceg

x	f	Δf	$\Delta^2 f$	$\Delta^3 f$	$\Delta^4 f$	$\Delta^5 f$	$\Delta^6 f$...
\vdots	\vdots							
x_{-4}	f_{-4}	\vdots						
		Δf_{-4}	\vdots					
x_{-3}	f_{-3}		$\Delta^2 f_{-4}$	\vdots				
			Δf_{-3}	$\Delta^3 f_{-4}$	\vdots			
x_{-2}	f_{-2}		$\Delta^2 f_{-3}$	$\Delta^3 f_{-3}$	$\Delta^4 f_{-4}$	\vdots		
			Δf_{-2}	$\Delta^3 f_{-2}$	$\Delta^4 f_{-3}$	$\Delta^5 f_{-4}$	\vdots	
x_{-1}	f_{-1}	Δf_{-1}	$\Delta^2 f_{-2}$	$\Delta^3 f_{-2}$	$\Delta^4 f_{-2}$	$\Delta^5 f_{-3}$	$\Delta^6 f_{-4}$	
x_0	f_0	Δf_0	$\Delta^2 f_{-1}$	$\Delta^3 f_{-1}$	$\Delta^4 f_{-1}$	$\Delta^5 f_{-2}$	$\Delta^6 f_{-3}$	
x_1	f_1		$\Delta^2 f_0$	$\Delta^3 f_0$	$\Delta^4 f_{-1}$	$\Delta^5 f_{-1}$	$\Delta^6 f_{-2}$...
			Δf_1					
x_2	f_2		$\Delta^2 f_1$	$\Delta^3 f_1$	$\Delta^4 f_0$	\vdots		
			Δf_2					
x_3	f_3		$\Delta^2 f_2$	\vdots				
			Δf_3					
x_4	f_4	\vdots						
\vdots	\vdots							

$$\begin{aligned}
 L_{2n+1}(x) = & f(x_0) + q \frac{\Delta f_{-1} + \Delta f_0}{2} + \frac{q^2}{2!} \Delta^2 f_{-1} + \frac{q(q^2 - 1)}{3!} \cdot \frac{\Delta^3 f_{-2} + \Delta^3 f_{-1}}{2} \\
 & + \dots + \frac{q(q^2 - 1) \cdot \dots \cdot (q^2 - n^2)}{(2n+1)!} \cdot \frac{\Delta^{2n+1} f_{-(n+1)} + \Delta^{2n+1} f_{-n}}{2}
 \end{aligned}$$

Aritmetička sredina prvog i drugog

x	f	Δf	$\Delta^2 f$	$\Delta^3 f$	$\Delta^4 f$	$\Delta^5 f$	$\Delta^6 f$...
\vdots	\vdots							
x_{-4}	f_{-4}	\vdots						
		Δf_{-4}	\vdots					
x_{-3}	f_{-3}		$\Delta^2 f_{-4}$	\vdots				
			Δf_{-3}	$\Delta^3 f_{-4}$	\vdots			
x_{-2}	f_{-2}		$\Delta^2 f_{-3}$	$\Delta^3 f_{-4}$	\vdots			
			Δf_{-2}	$\Delta^3 f_{-3}$	$\Delta^4 f_{-4}$	\vdots		
x_{-1}	f_{-1}		$\Delta^2 f_{-2}$	$\Delta^3 f_{-2}$	$\Delta^4 f_{-3}$	$\Delta^5 f_{-4}$	\vdots	
			Δf_{-1}	$\Delta^3 f_{-2}$	$\Delta^4 f_{-2}$	$\Delta^5 f_{-3}$	$\Delta^6 f_{-4}$	
x_0	f_0	Δf_0	$\Delta^2 f_{-1}$	$\Delta^3 f_{-1}$	$\Delta^4 f_{-2}$	$\Delta^5 f_{-2}$	$\Delta^6 f_{-3}$	
x_1	f_1	$\Delta^2 f_0$	$\Delta^3 f_0$	$\Delta^4 f_{-1}$	$\Delta^5 f_{-1}$	$\Delta^6 f_{-2}$	\vdots	
x_2	f_2	Δf_1	$\Delta^3 f_0$	$\Delta^4 f_0$	$\Delta^5 f_{-1}$	\vdots		
			$\Delta^2 f_1$					
x_3	f_3	Δf_2	$\Delta^3 f_1$	\vdots				
			$\Delta^2 f_2$					
x_4	f_4	Δf_3	\vdots					
\vdots	\vdots							

$$\begin{aligned}
 L_{2n+1}(x) = & \frac{f(x_0) + f(x_1)}{2} + \left(q - \frac{1}{2}\right)\Delta f_0 + \frac{q(q-1)}{2!} \cdot \frac{\Delta^2 f_{-1} + \Delta^2 f_0}{2} + \\
 & + \dots + \frac{q(q^2-1) \cdot \dots \cdot (q^2 - (n-1)^2)(q-n)(q-\frac{1}{2})}{(2n+1)!} \cdot \Delta^{2n+1} f_{-n}
 \end{aligned}$$