March 12, 2018

Centralne razlike

x	f	Δf	$\Delta^2 f$	$\Delta^3 f$	$\Delta^4 f$	$\Delta^5 f$	$\Delta^6 f$	...
$\vdots$	$\vdots$							
x_{-4}	f_{-4}	$\vdots$						
		Δf_{-4}	$\vdots$					
x_{-3}	f_{-3}		$\Delta^2 f_{-4}$	$\vdots$				
		Δf_{-3}		$\Delta^3 f_{-4}$	$\vdots$			
x_{-2}	f_{-2}		$\Delta^2 f_{-3}$		$\Delta^4 f_{-4}$	$\vdots$		
		Δf_{-2}		$\Delta^3 f_{-3}$		$\Delta^5 f_{-4}$	$\vdots$	
x_{-1}	f_{-1}		$\Delta^2 f_{-2}$		$\Delta^4 f_{-3}$		$\Delta^6 f_{-4}$	
		Δf_{-1}		$\Delta^3 f_{-2}$		$\Delta^5 f_{-3}$		
x_0	f_0		$\Delta^2 f_{-1}$		$\Delta^4 f_{-2}$		$\Delta^6 f_{-3}$	
		Δf_0		$\Delta^3 f_{-1}$		$\Delta^5 f_{-2}$		
x_1	f_1		$\Delta^2 f_0$		$\Delta^4 f_{-1}$		$\Delta^6 f_{-2}$	...
		Δf_1		$\Delta^3 f_0$		$\Delta^5 f_{-1}$	$\vdots$	
x_2	f_2		$\Delta^2 f_1$		$\Delta^4 f_0$	$\vdots$		
		Δf_2		$\Delta^3 f_1$	$\vdots$			
x_3	f_3		$\Delta^2 f_2$	$\vdots$				
		Δf_3	$\vdots$					
x_4	f_4	$\vdots$						
$\vdots$	$\vdots$							

$x \in (x_0, x_0 + \frac{h}{2}]$

Redosled čvorova: $x_0, x_1, x_{-1}, ..., x_n, x_{-n}, x_{n+1}$

$$\begin{aligned} L_{2n+1}(x) = f(x_0) + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_{-1}](x - x_0)(x - x_1) \\ + f[x_0, x_1, x_{-1}, x_2](x - x_0)(x - x_1)(x - x_{-1}) + ... \\ + f[x_0, x_1, ..., x_{-n}, x_{n+1}](x - x_0) \cdot ... \cdot (x - x_n)(x - x_{-n}) \end{aligned}$$

$$\begin{aligned} L_{2n+1}(x) = f(x_0) + q\Delta f_0 + \frac{q(q-1)}{2!}\Delta^2 f_{-1} + \frac{q(q^2-1)}{3!}\Delta^3 f_{-1} + ... \\ + \frac{q(q^2-1) \cdot ... \cdot (q^2 - n^2)}{(2n+1)!}\Delta^{2n+1} f_{-n} \end{aligned}$$

x	f	Δf	$\Delta^2 f$	$\Delta^3 f$	$\Delta^4 f$	$\Delta^5 f$	$\Delta^6 f$	...
$\vdots$	$\vdots$							
x_{-4}	f_{-4}	$\vdots$						
		Δf_{-4}	$\vdots$					
x_{-3}	f_{-3}		$\Delta^2 f_{-4}$	$\vdots$				
		Δf_{-3}		$\Delta^3 f_{-4}$	$\vdots$			
x_{-2}	f_{-2}		$\Delta^2 f_{-3}$		$\Delta^4 f_{-4}$	$\vdots$		
		Δf_{-2}		$\Delta^3 f_{-3}$		$\Delta^5 f_{-4}$	$\vdots$	
x_{-1}	f_{-1}		$\Delta^2 f_{-2}$		$\Delta^4 f_{-3}$		$\Delta^6 f_{-4}$	
		Δf_{-1}		$\Delta^3 f_{-2}$		$\Delta^5 f_{-3}$		
x_0	f_0		$\Delta^2 f_{-1}$		$\Delta^4 f_{-2}$		$\Delta^6 f_{-3}$	
		Δf_0		$\Delta^3 f_{-1}$		$\Delta^5 f_{-2}$		
x_1	f_1		$\Delta^2 f_0$		$\Delta^4 f_{-1}$		$\Delta^6 f_{-2}$	...
		Δf_1		$\Delta^3 f_0$		$\Delta^5 f_{-1}$	$\vdots$	
x_2	f_2		$\Delta^2 f_1$		$\Delta^4 f_0$	$\vdots$		
		Δf_2		$\Delta^3 f_1$	$\vdots$			
x_3	f_3		$\Delta^2 f_2$	$\vdots$				
		Δf_3	$\vdots$					
x_4	f_4	$\vdots$						
$\vdots$	$\vdots$							

$$x \in [x_0 + \frac{h}{2}, x_1)$$

Redosled čvorova: $x_1, x_0, x_2, x_{-1}, ..., x_{n+1}, x_{-n}$

$$\begin{aligned} L_{2n+1}(x) = & f(x_1) + f[x_1, x_0](x - x_1) + f[x_1, x_0, x_2](x - x_1)(x - x_0) \\ & + f[x_1, x_0, x_2, x_{-1}](x - x_1)(x - x_0)(x - x_2) + ... \\ & + f[x_1, x_0, ..., x_{n+1}, x_{-n}](x - x_1) \cdot ... \cdot (x - x_{-(n-1)})(x - x_{n+1}) \end{aligned}$$

$$\begin{aligned} L_{2n+1}(x) = & f(x_1) + (q-1)\Delta f_0 + \frac{q(q-1)}{2!}\Delta^2 f_0 + \frac{q(q-1)(q-2)}{3!}\Delta^3 f_{-1} + ... \\ & + \frac{q(q^2-1) \cdot ... \cdot (q^2-(n-1)^2)(q-n)(q-(n+1))}{(2n+1)!}\Delta^{2n+1} f_{-n} \end{aligned}$$

x	f	Δf	$\Delta^2 f$	$\Delta^3 f$	$\Delta^4 f$	$\Delta^5 f$	$\Delta^6 f$	...
$\vdots$	$\vdots$							
x_{-4}	f_{-4}	$\vdots$						
		Δf_{-4}	$\vdots$					
x_{-3}	f_{-3}		$\Delta^2 f_{-4}$	$\vdots$				
		Δf_{-3}		$\Delta^3 f_{-4}$	$\vdots$			
x_{-2}	f_{-2}		$\Delta^2 f_{-3}$		$\Delta^4 f_{-4}$	$\vdots$		
		Δf_{-2}		$\Delta^3 f_{-3}$		$\Delta^5 f_{-4}$	$\vdots$	
x_{-1}	f_{-1}		$\Delta^2 f_{-2}$		$\Delta^4 f_{-3}$		$\Delta^6 f_{-4}$	
		Δf_{-1}		$\Delta^3 f_{-2}$		$\Delta^5 f_{-3}$		
x_0	f_0		$\Delta^2 f_{-1}$		$\Delta^4 f_{-2}$		$\Delta^6 f_{-3}$	
		Δf_0		$\Delta^3 f_{-1}$		$\Delta^5 f_{-2}$		
x_1	f_1		$\Delta^2 f_0$		$\Delta^4 f_{-1}$		$\Delta^6 f_{-2}$	...
		Δf_1		$\Delta^3 f_0$		$\Delta^5 f_{-1}$	$\vdots$	
x_2	f_2		$\Delta^2 f_1$		$\Delta^4 f_0$	$\vdots$		
		Δf_2		$\Delta^3 f_1$	$\vdots$			
x_3	f_3		$\Delta^2 f_2$	$\vdots$				
		Δf_3	$\vdots$					
x_4	f_4	$\vdots$						
$\vdots$	$\vdots$							

$$x \in [x_0 - \frac{h}{2}, x_0)$$

Redosled čvorova: $x_0, x_{-1}, x_1, x_{-2}, x_2 \ldots, x_n, x_{-(n+1)}$

$$\begin{aligned} L_{2n+1}(x) = f(x_0) &+ f[x_0, x_{-1}](x - x_0) + f[x_0, x_{-1}, x_1](x - x_0)(x - x_{-1}) \\ &+ f[x_0, x_{-1}, x_1, x_{-2}](x - x_0)(x - x_{-1})(x - x_1) + \ldots \\ &+ f[x_0, x_{-1}, \ldots, x_n, x_{-(n+1)}](x - x_0) \cdot \ldots \cdot (x - x_{-n})(x - x_n) \end{aligned}$$

$$\begin{aligned} L_{2n+1}(x) = f(x_0) &+ q\Delta f_{-1} + \frac{q(q+1)}{2!}\Delta^2 f_{-1} + \frac{q(q^2-1)}{3!}\Delta^3 f_{-2} + \ldots \\ &+ \frac{q(q^2-1) \cdot \ldots \cdot (q^2-n^2)}{(2n+1)!}\Delta^{2n+1} f_{-(n+1)} \end{aligned}$$

x	f	Δf	$\Delta^2 f$	$\Delta^3 f$	$\Delta^4 f$	$\Delta^5 f$	$\Delta^6 f$	...
$\vdots$	$\vdots$							
x_{-4}	f_{-4}	$\vdots$						
		Δf_{-4}	$\vdots$					
x_{-3}	f_{-3}		$\Delta^2 f_{-4}$	$\vdots$				
		Δf_{-3}		$\Delta^3 f_{-4}$	$\vdots$			
x_{-2}	f_{-2}		$\Delta^2 f_{-3}$		$\Delta^4 f_{-4}$	$\vdots$		
		Δf_{-2}		$\Delta^3 f_{-3}$		$\Delta^5 f_{-4}$	$\vdots$	
x_{-1}	f_{-1}		$\Delta^2 f_{-2}$		$\Delta^4 f_{-3}$		$\Delta^6 f_{-4}$	
		Δf_{-1}		$\Delta^3 f_{-2}$		$\Delta^5 f_{-3}$		
x_0	f_0		$\Delta^2 f_{-1}$		$\Delta^4 f_{-2}$		$\Delta^6 f_{-3}$	
		Δf_0		$\Delta^3 f_{-1}$		$\Delta^5 f_{-2}$		
x_1	f_1		$\Delta^2 f_0$		$\Delta^4 f_{-1}$		$\Delta^6 f_{-2}$	...
		Δf_1		$\Delta^3 f_0$		$\Delta^5 f_{-1}$	$\vdots$	
x_2	f_2		$\Delta^2 f_1$		$\Delta^4 f_0$	$\vdots$		
		Δf_2		$\Delta^3 f_1$	$\vdots$			
x_3	f_3		$\Delta^2 f_2$	$\vdots$				
		Δf_3	$\vdots$					
x_4	f_4	$\vdots$						
$\vdots$	$\vdots$							

Aritmetička sredina prvog i treceg

x	f	Δf	$\Delta^2 f$	$\Delta^3 f$	$\Delta^4 f$	$\Delta^5 f$	$\Delta^6 f$	...
$\vdots$	$\vdots$							
x_{-4}	f_{-4}	$\vdots$						
		Δf_{-4}	$\vdots$					
x_{-3}	f_{-3}		$\Delta^2 f_{-4}$	$\vdots$				
		Δf_{-3}		$\Delta^3 f_{-4}$	$\vdots$			
x_{-2}	f_{-2}		$\Delta^2 f_{-3}$		$\Delta^4 f_{-4}$	$\vdots$		
		Δf_{-2}		$\Delta^3 f_{-3}$		$\Delta^5 f_{-4}$	$\vdots$	
x_{-1}	f_{-1}		$\Delta^2 f_{-2}$		$\Delta^4 f_{-3}$		$\Delta^6 f_{-4}$	
		Δf_{-1}		$\Delta^3 f_{-2}$		$\Delta^5 f_{-3}$		
x_0	f_0		$\Delta^2 f_{-1}$		$\Delta^4 f_{-2}$		$\Delta^6 f_{-3}$	
		Δf_0		$\Delta^3 f_{-1}$		$\Delta^5 f_{-2}$		
x_1	f_1		$\Delta^2 f_0$		$\Delta^4 f_{-1}$		$\Delta^6 f_{-2}$	...
		Δf_1		$\Delta^3 f_0$		$\Delta^5 f_{-1}$	$\vdots$	
x_2	f_2		$\Delta^2 f_1$		$\Delta^4 f_0$	$\vdots$		
		Δf_2		$\Delta^3 f_1$	$\vdots$			
x_3	f_3		$\Delta^2 f_2$	$\vdots$				
		Δf_3	$\vdots$					
x_4	f_4	$\vdots$						
$\vdots$	$\vdots$							

$$L_{2n+1}(x) = f(x_0) + q\frac{\Delta f_{-1} + \Delta f_0}{2} + \frac{q^2}{2!}\Delta^2 f_{-1} + \frac{q(q^2-1)}{3!} \cdot \frac{\Delta^3 f_{-2} + \Delta^3 f_{-1}}{2}$$
$$+ \ldots + \frac{q(q^2-1)\cdot\ldots\cdot(q^2-n^2)}{(2n+1)!} \cdot \frac{\Delta^{2n+1} f_{-(n+1)} + \Delta^{2n+1} f_{-n}}{2}$$

Aritmetička sredina prvog i drugog

x	f	Δf	$\Delta^2 f$	$\Delta^3 f$	$\Delta^4 f$	$\Delta^5 f$	$\Delta^6 f$	...
$\vdots$	$\vdots$							
x_{-4}	f_{-4}	$\vdots$						
		Δf_{-4}	$\vdots$					
x_{-3}	f_{-3}		$\Delta^2 f_{-4}$	$\vdots$				
		Δf_{-3}		$\Delta^3 f_{-4}$	$\vdots$			
x_{-2}	f_{-2}		$\Delta^2 f_{-3}$		$\Delta^4 f_{-4}$	$\vdots$		
		Δf_{-2}		$\Delta^3 f_{-3}$		$\Delta^5 f_{-4}$	$\vdots$	
x_{-1}	f_{-1}		$\Delta^2 f_{-2}$		$\Delta^4 f_{-3}$		$\Delta^6 f_{-4}$	
		Δf_{-1}		$\Delta^3 f_{-2}$		$\Delta^5 f_{-3}$		
x_0	f_0		$\Delta^2 f_{-1}$		$\Delta^4 f_{-2}$		$\Delta^6 f_{-3}$	
		Δf_0		$\Delta^3 f_{-1}$		$\Delta^5 f_{-2}$		
x_1	f_1		$\Delta^2 f_0$		$\Delta^4 f_{-1}$		$\Delta^6 f_{-2}$	...
		Δf_1		$\Delta^3 f_0$		$\Delta^5 f_{-1}$	$\vdots$	
x_2	f_2		$\Delta^2 f_1$		$\Delta^4 f_0$	$\vdots$		
		Δf_2		$\Delta^3 f_1$	$\vdots$			
x_3	f_3		$\Delta^2 f_2$	$\vdots$				
		Δf_3	$\vdots$					
x_4	f_4	$\vdots$						
$\vdots$	$\vdots$							

$$L_{2n+1}(x) = \frac{f(x_0) + f(x_1)}{2} + (q - \frac{1}{2})\Delta f_0 + \frac{q(q-1)}{2!} \cdot \frac{\Delta^2 f_{-1} + \Delta^2 f_0}{2} +$$
$$+ \ldots + \frac{q(q^2-1) \cdot \ldots \cdot (q^2 - (n-1)^2)(q-n)(q-\frac{1}{2})}{(2n+1)!} \cdot \Delta^{2n+1} f_{-n}$$